Chapter 2

Units and Measurements

1 UNITS

Measurement of any physical quantity involves comparison with certain basic arbitrarily chosen internationally accepted reference called units.

Classification

Fundamental units	Derived units
Independent of each other	Expressed as combination of fundamental units

- A complete set of these units, both the base units and derived units is known as system of units.
- Old system of units: CGS, FPS and MKS system.
- In **CGS** fundamental units are centimeter, gram and second.
- In **FPS** fundamental units are foot, pound and second.
- In **MKS** fundamental units are meter, kilogram and second.

2 SI SYSTEM OF UNITS (INTERNATIONAL SYSTEM OF UNITS)

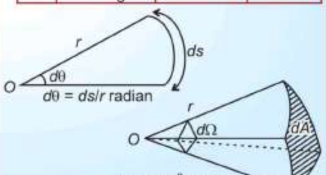
- Presently accepted internationally for measurement is SI system of units, revised in 2018. Certain rules to follow with standard symbols
- It is decimal system thus, conversion within system is easy and convenient
- It has 7 base unit and 2 supplementary units

Base Units

S.N.	Quantity	Unit	Symbol
1.	Length	meter	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Thermodynamic temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

Supplementary Units

S.N.	Quantity	Unit	Symbol
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr



$d\theta = ds/r$ radian

$d\Omega = dA/r^2$ steradian

3 MEASUREMENT OF LENGTH

- Large distance is measured by parallax method.
- Parallax angle $= \dfrac{\text{Basis}}{\text{Distance}}$
- $1° = 1.745 \times 10^{-2}$ rad
- $1'' = 4.85 \times 10^{-6}$ rad
- Measurement of very small distance like size of molecule uses, Optical microscope, Electronic microscope and Tunneling microscope
- 1 AU = 1.496×10^{11} m
- 1 ly = 9.46×10^{15} m
- 1 parsec = 3.08×10^{16} m
- Size of proton 10^{-15} m
- Radius of Earth 10^{7} m
- Distance to boundary of observable universe 10^{26} m

4 MEASUREMENT OF MASS

- SI unit is kilogram (kg)
- Unified atomic mass unit (u). It is used to measure mass of atoms and molecules
- 1 u = 1/12 × mass of one C-12 atom.
- 1 u = 1.66×10^{-27} kg
- Electron mass 10^{-30} kg
- Earth mass 10^{25} kg
- Observable universe 10^{55} kg

5 MEASUREMENT OF TIME

- Atomic standard of time: This is based on caesium clock, uncertainty gained overtime by caesium atomic clock is less than 1 part in 10^{13} (loss of 3 μs in one year)
- Time span of most unstable particle 10^{-24} s
- Travel time for light from nearest star 10^{8} s
- Age of universe 10^{17} s

6 ACCURACY and PRECISION

- Every measurement by any measuring instrument contains some uncertainty called error.
- Accuracy of a measurement is a measure of how close is the measured value to true value.
- Precision tells us to what resolution the quantity is measured.
- It is not necessary that more precise value is more accurate too.

7 ERRORS IN MEASUREMENT

Errors

- Systematic
  - Instrumental
  - Experimental
  - Personal
- Random

- Every measurement is approximate due to errors.
- Random errors occurs irregularly.
- Least count error is smallest value that can be measured by instrument (occurs within both systematic and random errors).
- Absolute error $= \dfrac{\Sigma(|a_i - a_{mean}|)}{n}$
- Relative error $= \dfrac{\Delta a_{mean}}{a_{mean}}$
- Percentage error $= \dfrac{\Delta a_{mean}}{a_{mean}} \times 100$

Combination of errors

- Sum and difference: $\Delta Z = \Delta A + \Delta B$
- Product or Quotient: $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

- If $X = \dfrac{A^a B^b}{C^c}$ then $\% \dfrac{\Delta x}{x} = a\left(\% \dfrac{\Delta A}{A}\right) + b\left(\% \dfrac{\Delta B}{B}\right) + c\left(\% \dfrac{\Delta C}{C}\right)$

8 SIGNIFICANT FIGURES

- Reliable digits plus first uncertain digit are known as significant digit.
- A choice of change of different units does not change number of significant digits.
- All non-zero digits are significant.
- All zero between two non-zero digits are significant.
- The terminal zeros in a number without a decimal point are not significant.
- The trailing zeros in a number with decimal point are significant.

Rules of Arithmatic Operations with Significant Figures

- Addition/Substraction: Final result contains as many decimal places as in number with least decimal places.
  e.g. 3.307 + 0.52 = 3.83
- Multiplication/Division: Result contains as many significant figures as in number with least number of significant figures.
  e.g. 4.11/1.2 = 3.4

Rounding off

- Preceding digit is raised by 1 if insignificant digit to be dropped is more than 5 and left unchanged of latter is less than 5.
- If insignificant digit is 5 then preceding digit is left unchanged if its even and uncreased by 1 if it is odd.

10 DIMENSIONAL FORMULAE AND SI UNITS OF VARIOUS PHYSICAL QUANTITIES

S. No.	Physical Quantity	Relation with other quantities	Dimensional Formula	SI Unit
1.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{M \times M} = [M^{-1}L^3T^{-2}]$	$N\,m^2\,kg^{-2}$
2.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	$N\,m^{-2}$
3.	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{ML^{-1}T^{-2}}{1} = [ML^{-1}T^{-2}]$	$N\,m^{-2}$
4.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	$N\,m^{-1}$
5.	Coefficient of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$	$N\,m^{-2}$ or Pa s or decapoise
6.	Planck's constant '*h*'	$\frac{E}{\nu} = \frac{\text{Energy}}{\text{Frequency}}$	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$	J s
7.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$	s^{-1}
8.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	$Pa\,m^{-1}$

9 DIMENSIONAL ANALYSIS

Dimensions

- Nature of physical quantity is determined by its dimension.
- The dimensions of physical quantity are powers to which base quantities are raised to represent it.
- The dimension of time in speed is –1.

Dimensional equation

- The expression which shows how and which of the base quantities represent the dimension of physical quantity is called dimensional formula.
- An equation is obtained by equating physical quantity with its dimensional formula.
- For example $[A] = [M^0L^2T^0]$

Homogeneity principle

Physical quantities represented by symbols on both sides of a mathematical equation must have same dimensions.

Applications

Checking dimensional consistency of equations

- It is based on homogeneity law. An equation is dimensionally correct if dimension of fundamental quantities of each term on left side of equation is equal to that on right hand side.

Deducing relations among physical quantities.

- We should know the dependence of physical quantity on other upto three physical quantities and product type of dependence

Limitations of Dimensional Analysis

- Dimensional analysis is useful in deducing relations among inter dependent physical quantities but dimensional constant can not be determined.
- It can test dimensional validity but not exact relationship between physical quantities having same dimensions.
- It does not distinguish between the physical quantities having same dimensions.

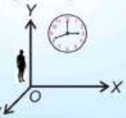
THE PHYSICS SPECIALIST

Motion in a Straight Line

3 Chapter

1 FRAME OF REFERENCE

- A rectangular coordinate system consisting of three mutually perpendicular axes, along with a clock. The point of intersection of these three axes is called origin (*O*)
- If a body changes its position as time passes w.r.t. frame of reference, it is said to be in motion.
- Motion of objects along a straight line is called rectilinear motion.



2 DISTANCE AND DISPLACEMENT

- **Distance:** Actual path length in motion. During motion it is non-zero
- **Displacement:** The shortest path between initial and final position. Equal to change in position. May or may not be equal to path length travelled. It can be positive, negative or zero.

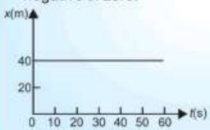


Fig: Stationary object

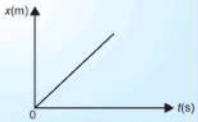


Fig: Object in uniform motion

3 SPEED

- The rate of distance covered with time is called speed,

$$v = \frac{\text{distance}}{\text{total time}} = \frac{\ell}{t}$$

Average Speed

$$v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{\text{total path length}}{\text{total time interval}}$$

Instantaneous speed

$$v = \lim_{\Delta t \to 0} \frac{\Delta \ell}{\Delta t} = \frac{d\ell}{dt}$$

4 VELOCITY

- The rate of change of position, It tells how fast position is changing with time and in what direction.

Average velocity

$$\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$$

- SI units are m s^{-1}

Instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

- Slope of position time graph

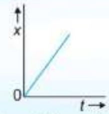


Fig: Moving with positive velocity

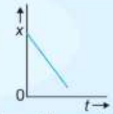


Fig: Moving with negative velocity

5 ACCELERATION

The time rate of change of velocity

Average Acceleration

$$\vec{a}_{av} = \frac{(v_2 - v_1)}{(t_2 - t_1)} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Uniform Acceleration

Equal change in velocity in equal intervals of time

Non-Uniform Acceleration

Acceleration changes with time

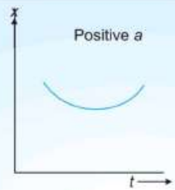


Fig: Positive acceleration

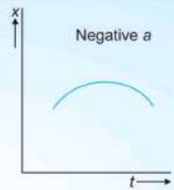


Fig: Negative acceleration

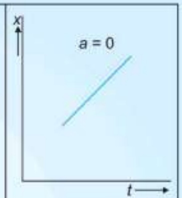


Fig: Zero acceleration

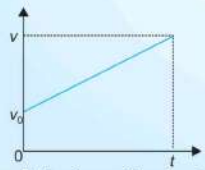


Fig: Motion in positive direction with positive acceleration

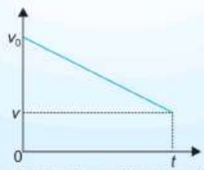


Fig: Motion in positive direction with negative acceleration

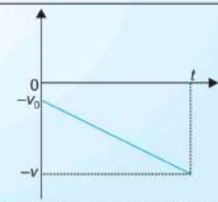
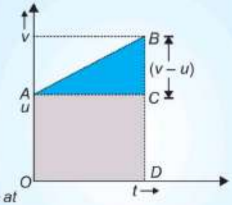


Fig: Motion in negative direction with negative acceleration

6 Kinematic Equations

- A mathematical treatment to describe the motion of a body in one-dimension.

For uniformly accelerated motion



- $v = u + at$
- $s = ut + \frac{1}{2}at^2 = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s_n = u + \frac{a}{2}(2n-1)$
- $\bar{v} = \frac{u+v}{2}$

7 FOR MOTION WITH VARIABLE ACCELERATION

- $\frac{dv}{dt} = a \Rightarrow v - u = \int a dt$
- $\frac{dx}{dt} = v \Rightarrow \Delta x = \int v dt$ (Area under $v-t$ curve)
- $\frac{v dv}{dx} = a$
- $\frac{d^2x}{dt^2} = a$

9 Relative Velocity

- The velocity with which an object moves with respect to another object is called relative velocity.

$v_{AB} = (v_A - v_B)$

$v_{AB} = (v_A + (-v_B))$

$v_{BA} = (v_B - v_A)$

8 FOR MOTION UNDER GRAVITY

- A mathematical treatment to describe the motion of a body in one-dimension under free fall

Vertically downward motion

When object is released from $y = 0$

- $v = -gt$
- $y = -\frac{1}{2}gt^2$
- $v^2 = -2gy$

Vertically upward motion

$u \neq 0$, acceleration $a = -g$

- $v = u - gt$
- $S = ut - \frac{1}{2}gt^2$
- $v^2 = u^2 - 2gh$

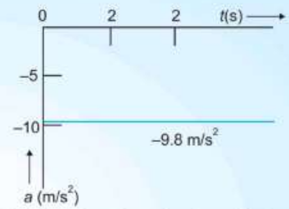


Fig: Variation of acceleration with time

- Distance travelled during equal intervals of time by a body falling freely from rest is in ratio 1 : 3 : 5 : 7 : 9 : 11 .. (Galileo's law)

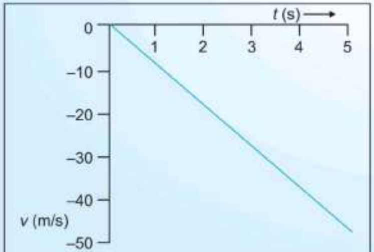


Fig: Variation of velocity with time

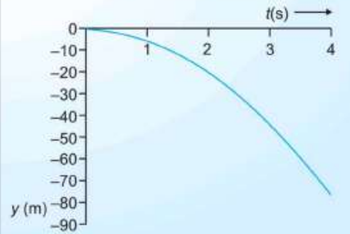


Fig: Variation of distance with time

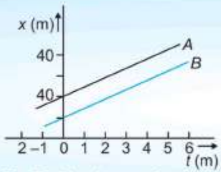


Fig: Position-time graphs of two objects with equal velocities

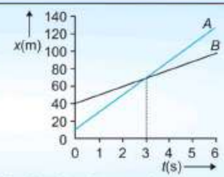


Fig: Position-time graphs of two objects with unequal velocities, showing the time of meeting

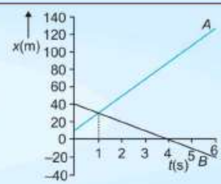


Fig: Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting

Motion in a Plane

THE PHYSICS SPECIALIST

Chapter 4

1 SCALARS AND VECTORS

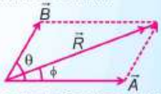
- **Scalar quantity:** It has only magnitude with proper unit. All base quantities are scalar. The rules combining scalars are rules of ordinary algebra.
- **Vector quantity:** It has both magnitude and direction and obeys the triangle law or parallelogram law of vector addition.
- **Equality of vector:** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal, if and only if, they have same magnitude and direction.
- **Multiplication of vector by real numbers:** If a vector $\vec{A}$ is multiplied by real number λ, then $A' = \lambda|\vec{A}|$

  if $\lambda > 0$, magnitude will change and direction remains same

  if $\lambda < 0$, magnitude changes λ times and direction gets reverse.
- **Parallelogram law of vector addition:** For two co-initial vectors represented by two adjacent sides of a parallelogram, the diagonal of a parallelogram passing through same point will be resultant.

  $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

  $\tan\phi = \dfrac{B\sin\theta}{A + B\cos\theta}$

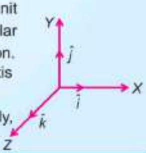


- **Subtraction of vector:** It can be defined as addition of a vector and negative of other vector.

  $\vec{S} = \vec{A} - \vec{B}$

  $\vec{S} = \vec{A} + (-\vec{B}) \Rightarrow |\vec{S}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

Unit Vectors: It is a vector of unit magnitude and points in a particular direction. It has no unit and dimension. Unit vectors along the x, y and z axis of a rectangular coordinate system represented by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively, called basic unit vectors.



2 RESOLUTION OF VECTORS

$\vec{A} = \overline{OP} = \overline{OQ} + \overline{QP}$

$\vec{A} = \lambda\vec{a} + \mu\vec{b}$

3 RECTANGULAR COMPONENTS

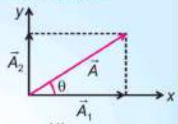
- $\vec{A} = \vec{A}_1 + \vec{A}_2$

  $\vec{A} = A_x\hat{i} + A_y\hat{j}$

  $\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$

  $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

  $\tan\theta = \dfrac{A_y}{A_x}$, $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$

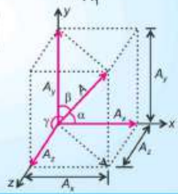


- Resolution in three rectangular components

  $A_x = A\cos\alpha$, $A_y = A\sin\alpha$

  $A_z = A\cos\gamma$

  $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



4 MOTION IN A PLANE

$\vec{r} = x\hat{i} + y\hat{j}$

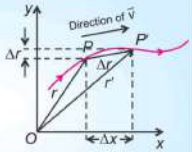
$\vec{r}' = x'\hat{i} + y'\hat{j}$

$\overline{\Delta r} = \vec{r}' - \vec{r}$

$\overline{\Delta r} = (x' - x)\hat{i} + (y' - y)\hat{j}$

$\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$

Instantaneous velocity, $\vec{v} = \dfrac{d\vec{r}}{dt}$



- The direction of velocity at any point on path is tangent to path and in direction of motion.

5 MOTION IN A PLANE WITH CONSTANT ACCELERATION

- $\vec{v} = \vec{v}_o + \vec{a}t$

  $v_x = v_{ox} + a_x t$

  $v_y = v_{oy} + a_y t$

- $\vec{r} = \vec{r}_o + \vec{v}_o t + \dfrac{1}{2}\vec{a}t^2$, $x = x_o + v_{ox}t + \dfrac{1}{2}a_x t^2$

  $y = y_o + v_{oy}t + \dfrac{1}{2}a_y t^2$

6 RELATIVE VELOCITY IN TWO DIMENSIONS

The velocity of object A relative to B

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

where $\vec{V}_A$ and $\vec{V}_B$ are velocities in the same frame.

Similarly, $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\vec{V}_{AB} = -\vec{V}_{BA}$ and $|\vec{V}_{AB}| = |\vec{V}_{BA}|$

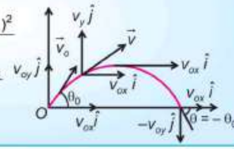
7 PROJECTILE MOTION

Equation of trajectory $y = x\tan\theta_0 - \dfrac{1}{2}\dfrac{gx^2}{v_0^2\cos^2\theta_0}$

This is equation of parabola.

- Time of flight $T_f = \dfrac{2v_o\sin\theta_0}{g}$
- Maximum height $h_m = \dfrac{(v_o\sin\theta_0)^2}{2g}$
- Horizontal range $R = \dfrac{v_o^2\sin 2\theta_0}{g}$

for R_{max}, $\theta = 45°$ $R_{max} = \dfrac{v_o^2}{g}$

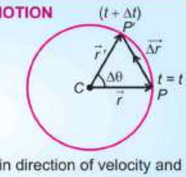


8 UNIFORM CIRCULAR MOTION

In uniform circular motion particle moves with constant speed.

- Angular displacement $\Delta\theta = \dfrac{\text{Arc }(PP')}{r}$
- Angular velocity $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi}{T} = 2\pi\nu$
- Linear speed $v = r\omega$
- Centripetal acceleration-Due to change in direction of velocity and is always directed towards centre.

  $a = \dfrac{v^2}{r} = r\omega^2 = 4\pi^2\nu^2 r = v\omega$



Laws of Motion

1 NEWTON'S 1ST LAW

A body continues its state of rest or of motion until unless an external force is acted on it

Inertia of rest

The property of body due to which it cannot change its state of rest by itself.

Inertia of motion

The property of body due to which it cannot change its state of motion by itself.

Inertia of direction

The property due to which a body cannot change its direction of motion by itself.

2 NEWTON'S 2ND LAW

The rate of change of Linear momentum of a body is directly proportional to the external force applied on the body and takes place in the direction in which force acts

$F = \frac{dp}{dt} = ma$

- The same force for the same time causes same change in momentum for different bodies.

Impulse

A large force acts for very short duration of time produces a finite change in momentum. Product of force and time duration for which it acts is impulse.

Impulse = $F \times \Delta t = \Delta p$

Equilibrium of a particle

$\Sigma \vec{F} = 0 \Rightarrow \Sigma F_x = 0, \Sigma F_y = 0$ and $\Sigma F_z = 0$

Conservation of Liner Momentum

Total momentum of an isolated system of intracting particles is conserved if there is no external force acting on it.

$\vec{p}_{initial} = \vec{p}_{final}$

4 NON-INTERTIAL FRAME OF REFERENCE

Pseudo Force $\vec{F}_{pseudo} = -M\vec{a}_{frame}$

$\vec{F}_{ext} + \vec{F}_{pseudo} = M\vec{a}$

3 NEWTON'S 3RD LAW

To every action there is always an equal and opposite reaction

$\vec{F}_{AB} = -\vec{F}_{BA}$

- Forces always occur in pairs. Force on body A by B is equal and opposite to force on body B by A.

Some examples of Newton's 3rd Law

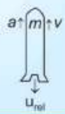
- Recoiling of Gun
- Rowing of boat
- When a man jumps from a boat, the boat moves backward
- It is difficult to walk on sand or ice.

Rocket Propulsion

- $a = \frac{u_{rel}}{m}\frac{dm}{dt} - g$

Thrust

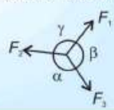
- $F = -u_{rel}\frac{dm}{dt}$



7 PROBLEM SOLVING TECHNIQUES IN MECHANICS

- Identify the unknown forces and accelerations
- Draw FBD of bodies in system
- Resolve forces into components
- Apply $\Sigma \vec{F} = 0$ in the direction of equilibrium
- Apply $\Sigma \vec{F} = M\vec{a}$ in the direction of accelerated motion
- Write constraint relations if exists.
- Solve the equations $\Sigma \vec{F} = 0$ and $\Sigma \vec{F} = M\vec{a}$
- For equilibrium of concurrent forces use sine rule

$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$



5 COMMON FORCES IN MECHANICS

Tension Force

- Restoring force in string is called tension.
- It is due to electromagnetic force
- Always acts away from the body
- It is a contact force.

Weight

- It is equal to the gravitational pull *i.e.* $W = Mg$
- It is non-contact force.

Normal Reaction

It is always perpendicular to the surface in contact.

- It is a contact force.

Spring Force

- $\vec{F} = -K\vec{x}$
- It is due to electromagnetic force
- It is a contact force.

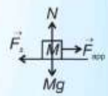
Friction

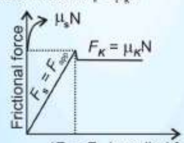
It is the resistance offered to the relative motion between two bodies in contact

- It is parallel to surface of body in contact.

Type of Friction

- Static friction: $F_s = F_{applied}$
- Limiting friction $F_{lim} = \mu_s N$
- Kinetic friction $F_k = \mu_k N$





- Acceleration of body sliding down a rough inclined plane
  $a = g(\sin\theta - \mu\cos\theta)$
- Angle of friction: $\theta = \tan^{-1}(\mu_s)$
- Angle of repose: $\alpha = \tan^{-1}(\mu_s)$

6 CIRCULAR MOTION

A body moving in a circular path is called circular motion.

$F_c = mv^2/R$ is called centripetal force.

Uniform circular motion

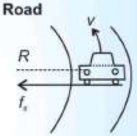
- $a = a_c = \frac{v^2}{R} = R\omega^2$
- $a = a_c = v.\omega$

Non-uniform circular motion

- $\vec{a} = \vec{a}_T + \vec{a}_c$
- $a = \sqrt{a_T^2 + a_c^2}$

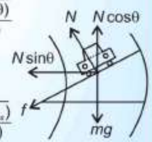
Motion of car on level Road

- $v_{max} = \sqrt{\mu_s Rg}$
- $\mu_{min} = \frac{v^2}{Rg}$
- $R_{min} = \frac{v^2}{\mu g}$



Motion of car on Banked Road

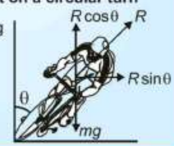
- $v_{max} = \sqrt{\frac{Rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}}$
- $v_{optimum} = \sqrt{Rg\tan\theta}$
- $v_{min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{(1 + \mu_s\tan\theta)}}$



Bending of cyclist on a circular turn

- Angle of Bending

$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$



- Numerically: $\alpha = \theta$
- Kinetic friction is usually less than maximum value of static friction.

Work, Energy and Power

Chapter 6

1 SCALAR PRODUCT

- Also called dot product
- $\vec{A}\cdot\vec{B} = AB\cos(\theta)$
- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
- $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- $(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}) = A_xB_x + A_yB_y = A_zB_z$

2 WORK

- Scalar product of force and displacement is work.
- Work done by a force can be positive, negative or zero.
- Work done by gravity in horizontal displacement of object is zero.
- Work done by tension in pendulum bob is zero.
- Work done by spring elastic force during stretching or compressing is negative.
- Work can be done by a constant or variable force.

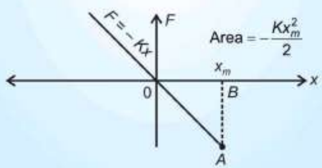
A. Constant force $W = \vec{F}\cdot\Delta\vec{r}$

B. Variable force $W = \int_{r_1}^{r_2}\vec{F}(r)\cdot d\vec{r}$

C. When force-displacement graph is given, Area under force-displacement curve gives work done by the force.

D. Work done in stretching a spring by distance Δl is

$W = \frac{1}{2}K(\Delta l)^2$



(Potential energy) $U = -W_S = \frac{1}{2}kx_m^2$

3 ENERGY

- Capability of an object to perform work is its energy.

Some common units of energy

Kilowatt hour	3.6×10^6 J
erg	10^{-7} J
Electron volt	1.6×10^{-19} J
Calorie	4.186 J

Mechanical energy is of two forms

Kinetic Energy

- Energy a body possesses by virtue of its motion.
- $KE = \frac{1}{2}mv^2$
- Its unit is joule
- Work is related to KE of body by theorem called work-energy theorem.

$$\Delta K = K_f - K_i = W = \int_{x_1}^{x_2} F(x)\,dx$$

- This theorem is in scalar form.
- Shape of graph between KE of a body and its speed is parabola.
- $(F_c + F_{nc})\Delta x = \Delta K$, when both forces are present

  F_c = Conservative forces

  F_{nc} = Non-conservative forces
- Kinetic energy of a body of fixed mass is directly proportional to square of its momentum.

$$K = \frac{P^2}{2m}$$

- Kinetic energy of fast moving air is used to generate electricity in wind mill.
- If two objects have same momentum, then the lighter has more kinetic energy and vice versa.
- Kinetic energy of fast flowing stream has been used to grind corn and now to generate hydro-electricity.

Potential Energy

- It is form of stored energy, by virtue of position or configuration of body.
- Notion of potential energy is applicable to class of conservative forces. Work done against such forces gets stored up as potential energy. When constraints are removed, this energy may appear as kinetic energy.
- Change in potential energy for a conservative force; ΔU is equal to negative of work done by the force
- $\Delta U = -\int_{\infty}^{r}\vec{F}\cdot d\vec{r}$
- A force is conservative if it is derived from a scalar quantity $U(x)$ by relation $F_x = -\frac{dU}{dx}$
- Work done by a conservative force depends only on initial and final points. Zero of potential energy is arbitrary.
- Work done by gravity depends on initial and final position only $U_h = mgh$ (Gravitational potential energy at height h)
- Potential energy of a stretched spring

$$U = \frac{Kx_m^2}{2}$$

K is spring constant. Spring is said to be stiff if K is high, x_m = extension of spring.

4 LAW OF CONSERVATION MECHANICAL ENERGY

- If conservative and non-conservative forces acts on a body then

  $(F_c + F_{nc})\Delta x = \Delta K$

  Now, $F_c \Delta x = -\Delta U$

  $\Delta(K+U) = F_{nc}\Delta x$

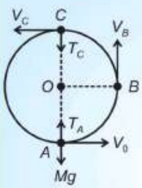
  $\boxed{\Delta E = F_{nc}\Delta x}$

  E = Total Mechanical Energy (Consequence of work energy theorem). If $F_{nc} = 0$ then $\boxed{\Delta E = 0}$

- Mechanical energy of a system is conserved if the forces doing work on it are conservative.

Conservative Forces	Non-conservative Forces
The work done by or against the force in moving a body depends only on initial and final position of the body and not on path followed in between.	The work done by or against the force in moving a body from one position to another depends on the path followed between the initial and final positions.

5 VERTICAL CIRCULAR MOTION



$T_A = \frac{MV_0^2}{L} + Mg$ and $E_A = \frac{1}{2}MV_0^2$

$E_C = \frac{1}{2}MV_C^2 + 2MgL$ = constant

$Mg = \frac{MV_C^2}{L}$ when string slackens (just completes loop)

$E_A = E_C = \frac{5}{2}MgL = \frac{MV_0^2}{2}$

$\therefore V_0 = \sqrt{5gL}$

Minimum speed at different locations to complete loop $V_C = \sqrt{gL}$

$V_B = \sqrt{3gL}$

$K_A : K_B : K_C = 5 : 3 : 1$

7 COLLISION

- Exchange of momentum between objects is consequence of collision, due to material impulsive forces. The laws of momentum andenergy conservation are used in collision.
- Collision are classified as elastic and inelastic collision depending on nature of colliding bodies.
- In all collisions, total linear momentum of the systemis conserved. Initial momentum of system is equal to final momentum of the system.

COLLISION → PERFECT INELASTIC COLLISION / INELASTIC / ELASTIC COLLISION

PERFECT INELASTIC COLLISION

A collision in which two colliding particles move together (in one dimensions) after the collision is complete inelastic collision. Kinetic energy is always lost in such collisions.

For m_1 moving at u_1 and m_2 at rest.

$$v_f = \frac{m_1u_1}{m_1 + m_2}$$

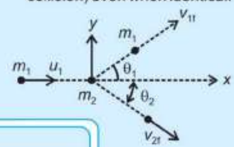
Loss in KE on collision

$$\boxed{\Delta K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1)^2}$$

INELASTIC

An intermediate collision case where the deformation is partly restored and some of initial kinetic energy is lost. Momentum of two colliding bodies before and after will remain conserved.

- In two dimension, inelastic collision if target at rest, two object don't move at right angles to each other (glancing collision) even when identical.



ELASTIC COLLISION

Both linear momentum and kinetic energy of system of colliding particles will remain conserved.

- In one dimensional collision with target initially at rest

  $$\boxed{v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1}$$

  $$\boxed{v_{2f} = \frac{2m_1u_1}{m_1 + m_2}}$$

- If two masses are equal and target is at rest

  $\boxed{v_{1f} = 0,\ v_{2f} = u_1}$

  First one comes to rest and pushes off the second with its initial speed. Thus, velocities are exchanged

- If $m_2 >> m_1$

  $\boxed{v_{1f} = -u_1 \text{ and } v_{2f} \simeq 0}$

  Heavier mass is undisturbed while lighter mass reverses its velocity.

- Fraction of kinetic energy lost by targetting body when target at rest

  $$f_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

  and fraction of KE gained by target being at rest initially

  $$f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

- When two equal masses undergo two dimensional elastic collision with one of them at rest, after collision they will move at right angles to each other.

6 POWER

- Rate at which work is done is power.
- $P = \frac{dW}{dt} = F \cdot \frac{dr}{dt} = \vec{F} \cdot \vec{v}$
- Rate at which energy is transferred is power.
- Average power is ratio of total work to total time taken.
- $P_{av} = \frac{W}{t}$
- SI unit of power is watt.
- Another unit of power is horse power.

  [1 hp = 746 W]
- A machine which performs same amount of work over a shorter period of time has more power.

System of Particles and Rotational Motion

7 Chapter

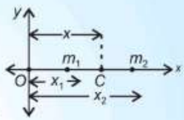
1 RIGID BODY

- Ideally a rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.
- In pure translational motion at any instant of time all particles of the body have same velocity.
- The motion of rigid body which is pivoted or fixed is rotation. Every particle of the body moves in a circle.
- The motion of rigid body which is not pivoted or fixed in some way is either a pure translation or is combination of translation and rotation

2 CENTRE OF MASS

- COM is an imaginary point where mass of an extended body is assumed to be concentrated
- This concept is used to study indepedently translatory and rotatory motion under effect of external forces.
- The laws of motion which are applied to particles can be applied to large sized bodies by converting body into a particle at location of COM.
- **Centre of mass for two particle system**

$$\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$



- For x and y plane

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \text{ and, } Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

- For a system of n particles distributed in space,

$$X_{cm} = \frac{\sum m_i x_i}{M}, Y_{cm} = \frac{\sum m_i y_i}{M}, Z_{cm} = \frac{\sum m_i z_i}{M}$$

- **COM For Continous Mass**

If the body has continous distribution of mass (RING, DISC, ROD)

$$\boxed{\vec{R} = \frac{1}{M}\int \vec{r}dm} \quad M = \text{total mass of body}$$

The co-ordinates of COM of body,

$$X_{cm} = \frac{1}{M}\int xdm, Y_{cm} = \frac{1}{M}\int ydm, Z_{cm} = \frac{1}{M}\int zdm$$

- If we choose centre of mass at origin $\int \vec{r}dm = 0, \int xdm = \int ydm = \int zdm = 0$
- For homogeneous bodies of regular shape, centre of mass lies at geometric centre.

3 MOTION OF COM

$$M\vec{R} = \sum m_i\vec{r}_i$$

$$\therefore \ M\vec{V} = \sum m_i\vec{v}_i$$

- Velocity of COM of system

$$\boxed{\therefore \ \vec{V} = \frac{\sum m_i\vec{v}_i}{M}}$$

- Acceleration Of Com of System

$$\vec{A} \text{ or } \vec{a}_{cm} = \frac{\sum m_i\vec{a}_i}{M}$$

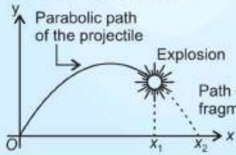
- Total mass of system of particles times the acceleration of its centre of mass is vector sum of all forces acting on system of particles.

$$M\vec{A} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n$$

$$M\vec{A} = \vec{F}_{ext}$$

$$\boxed{\vec{A} = \frac{\vec{F}_{ext}}{M} = \frac{\text{Total external force}}{\text{Total mass of system}} = \frac{\sum m_i\vec{a}_i}{M}}$$

- Centre of mass of the system of particles moves as if all mass of a system was concentrated at centre of mass and all the external forces were applied at that point.
- A projectile following parabolic path explodes into fragments in mid air. The forces leading to explosion are internal, they contribute nothing to motion of COM. Total external force gravity acting on body is same before and after explosion. The COM under influence of external forces continue along same parabolic trajectory as it would have followed without explosion.



4 LINEAR MOMENTUM OF SYSTEM OF PARTICLES

- Velocity of COM for a system of n particles

$$\vec{P} = \vec{p}_1 + \vec{P}_2 + ...+ \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + ...+ m_n\vec{v}_n$$

$$\boxed{\vec{V} = \frac{m_1\vec{v}_1 + m_1\vec{v}_2 + ... + m_n\vec{v}_n}{M}}$$

This is the velocity of centre of mass

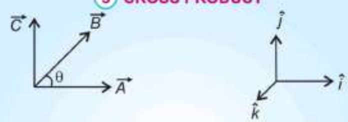
- Total linear momentum of system of particles is equal to the product of total mass of system and velocity of its centre of mass.
- When total external force acting on a system of particles is zero, total linear momentum of system is constant. The velocity of centre of mass remains constant.

$$\vec{P} = m\vec{v}$$

$$\text{if } \vec{F}_{ext} \Rightarrow \frac{d\vec{P}}{dt} = 0 \quad \boxed{P = \text{constant}}$$

- If centre of mass was initially at rest, for no external force, centre of mass will remain at rest.

5 CROSS PRODUCT



- $\vec{C} = \vec{A} \times \vec{B}$

$|\vec{C}| = |\vec{A}||\vec{B}|\sin\theta$

θ is angle between $\vec{A}$ and $\vec{B}$

- Properties

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$

$\vec{A} \times \vec{A} = \vec{0}$

$\hat{i} \times \hat{i} = \vec{0}$

$\hat{j} \times \hat{j} = \vec{0}$

$\hat{k} \times \hat{k} = \vec{0}$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

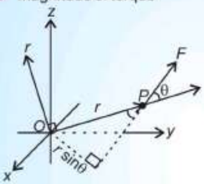
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{j} \times \hat{k} = -\hat{i}$

(6) MOMENT OF FORCE (TORQUE)

- Analogue of force in case of rotational motion is torque, which is turning effect of a force.
- $\vec{\tau} = \vec{r} \times \vec{F}$ when force acts on a particle whose position vector w.r.t. origin is $\vec{r}$.
- This is a vector quantity having SI units N m.
- Magnitude of torque



$\tau = r F \sin\theta$

$\tau = (r\sin\theta) \times F = r_{\perp} F$

$\tau = rF\sin\theta = rF_{\perp}$

$r_{\perp} = r\sin\theta$ = perpendicular distance of line of action of force from origin (axis of rotation) and $\vec{F}_{\perp}$ is component of $\vec{F}$ Perpendicular to $\vec{r}$.

- If direction of $\vec{r}$ and $\vec{F}$ are reversed, the direction of moment of force remains same.
- Couple : A pair of equal and opposite forces with different lines of action is known as a couple. A couple produces rotation without translation example : opening a bottle.

(8) EQUILIBRIUM OF RIGID BODY

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time or equivalently, the body has neither linear acceleration nor angular acceleration.

- Vector sum of forces on rigid body is zero $\sum \vec{F}_i = 0$
- Vector sum of torques on rigid body is zero. $\sum \vec{\tau}_i = 0$
- Rotational equilibrium condition is independent of location of origin about which torques are taken.
- A body may be in partial equilibrium i.e. rotational equilibrium but not transnational.

(7) ANGULAR MOMENTUM

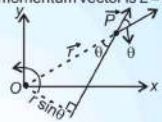
- It is referred as moment of linear momentum. For a particle,

$\vec{L} = \vec{r} \times \vec{P}$

The magnitude of angular momentum vector is $L = rp\sin\theta$

$L = r \times P\sin\theta = r \times P_{\perp}$

$L = r\sin\theta \times P = r_{\perp} \times P$



$r_{\perp} = (r\sin\theta)$ = is perpendicular distance of directional line of $\vec{P}$ from origin and

$P_{\perp}$ = component of P in the direction perpendicular to $\vec{r}$

- Angular momentum will be zero when $P = 0$ or particle is at origin or or line of P passes through origin.
- **Angular Momentum Conservation Law**

$\vec{\tau} = \frac{d\vec{L}}{dt}$ (time rate of change of angular momentum of a system of particles is equal to torque acting on it.)

- If total external torque on a system of particles is zero, total angular momentum remains constant for the system.

$\vec{\tau} = 0 \rightarrow \frac{d\vec{L}}{dt} = 0$

$\vec{L}$ = constant

(9) MOMENT OF INERTIA : MOI

- Analogue of mass, in rotational motion is rotational inertia also called moment of inertia.
- This is a characteristics of rigid body and the axis about which it rotates. It depends on distribution of mass and position of axis of rotation.
- This parameter is independent of magnitude of angular velocity of body, For a system of particles moment of inertia is given by $I = \sum_{i=1}^{n} m_i r_i^2$

MOMENT OF INERTIA OF DIFFERENT RIGID BODIES

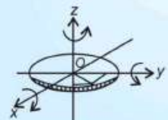
(Regular Shaped)

Body	Axis	I
Thin circular ring, radius R	Perpendicular to plane, at centre	MR^2
Thin circular ring, radius R	Diameter	$MR^2/2$
Thin rod, length L	Perpendicular to rod, at mid point	$ML^2/12$
Circular disc, radius R	Perpendicular to disc at centre	$MR^2/2$
Circular disc, radius R	Diameter	$MR^2/4$
Hollow cylinder, radius R	Axis of cylinder	MR^2
Solid cylinder, radius R	Axis of cylinder	$MR^2/2$
Solid sphere, radius R	Diameter	$2MR^2/5$
Hollow sphere, radius R	Diameter	$2MR^2/3$

THEOREMS OF MOI

Theorem of perpendicular axes

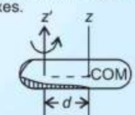
- Theorem is applicable to bodies whose thickness is small compared to other dimensions. (Planar body)
- MOI of a planar body about an axis perpendicular to its plane is equal to the sum of its MOI about two perpendicular axes concurrent with perpendicular axis and lying in plane of body.



$I_z = I_x + I_y$

Theorem of parallel axes

- The theorem is applicable to body irrespective of any shape.
- MOI of a body about any axis is equal to the sum of MOI of the body about a parallel axis passing through its COM and the product of its mass and the square of distance between the two parallel axes.



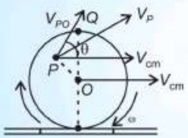
$I_{z'} = I_z + md^2$

(10) TRANSNATIONAL AND ROTATIONAL MOTION ANALOGY

Linear motion	Rotation about a fixed Axis
Displacement x	Angular displacement θ
Velocity $v = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Mass m	Moment of inertia I
Force $F = ma$	Torque $\tau = I\alpha$
Work $dW = Fds$	Work $= dW = \tau d\theta$
Kinetic energy $k = \frac{mv^2}{2}$	Kinetic energy $k = \frac{I\omega^2}{2}$
Power $P = \vec{F}.\vec{V}$	Power $P = \vec{\tau}.\vec{\omega}$
Linear momentum $P = mV$	Angular momentum $L = I\omega$
$\vec{F} = \frac{d\vec{p}}{dt}$	$\vec{\tau} = \frac{d\vec{L}}{dt}$

(11) ROLLING MOTION

- All wheels used in transportation have rolling motion.
- It is combination of rotation and translation with axis moving.
- When disc rolls without slipping, At any instant of time bottom of disc which is in contact with surface is at rest with respect to surface.



$$\vec{V}_P = \vec{V}_{PO} + \vec{V}_{cm}$$

$$|V_P| = [(V_{PO})^2 + V_{cm}^2 + 2V_{PO}V_{cm}\cos\theta]^{1/2}$$

- In pure rolling with out slipping $\Rightarrow v = R\omega$
- Top of a rolling body has magnitude of velocity

$V_Q = V_{cm} + \omega_{cm}R = V_{cm} + V_{cm} = 2\,V_{cm}$, Bottom is at rest w.r.t. surface

(12) KINETIC ENERGY OF TRANSLATING AND ROTATING BODIES:

- K.E of translation + K.E of rotational motion

$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2$$

where $I = MK^2$, K is corresponding radius of gyration

Radius of Gyration : Distance from axis of rotation of a point mass whose mass is equal to mass of whole body and whose moment of inertia is equal to moment of inertia of body about the axis.

- **Kinetic Energy In Case Of Pure Rolling Motion**

$$V_{cm} = R\omega_{cm}$$

$$KE = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}MK^2\left(\frac{V_{cm}^2}{R^2}\right)$$

$$KE = \frac{1}{2}MV_{cm}^2\left[1 + \frac{K^2}{R^2}\right]$$

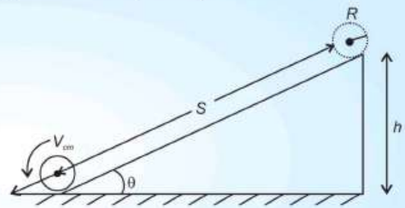
This formula can be used to all rolling bodies like ring, disc, cylinder sphere.

(13) KINEMATICS OF ROLLING BODIES DOWN ROUGH INCLINE PLANE

We apply conservation of mechanical energy to rolling bodies as Rolling friction performs no work.

$$\Delta P.E. = \Delta K.E.$$

$$mgh = \frac{1}{2}mV_{cm}^2\left[1 + \frac{K^2}{R^2}\right]$$



Velocity at bottom $V_{cm} = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$

Acceleration of COM ; $a = \dfrac{g\sin\theta}{1 + (K/R)^2}$

Minimum coefficient of friction required for pure rolling $\mu = \left(\dfrac{K^2}{R^2 + K^2}\right)\tan\theta$

Time to reach the bottom $= \dfrac{1}{\sin\theta} \times \sqrt{\dfrac{2h}{g} \times \left(\dfrac{K^2}{R^2} + 1\right)}$

Gravitation

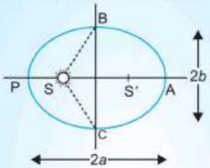
THE PHYSICS SPECIALIST

8 Chapter

1 KEPLER'S LAWS OF PLANETARY MOTION

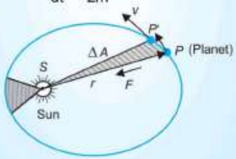
Law of Orbits

Every planet revolves around the sun in an elliptical orbit and the sun in situated at one of its foci.



Law of Areas

The areal velocity of the planet around the sun is constant i.e. $\frac{dA}{dt} = \frac{L}{2m}$ = constant



Law of Periods

The square of the time period of revolution of a planet is directly proportional to the cube of semi major axis length of the elliptical orbit *i.e.* $T^2 \propto a^3$

4 VARIATION OF ACCELERATION DUE TO GRAVITY (g)

Due to Altitude (h)

The value of g goes on decreasing with height (h)

$$g_h = \frac{GM_e}{(R_e+h)^2}$$

2 NEWTON'S LAW OF GRAVITATION

- The Gravitational force (F) between two bodies is directly proportional to product of masses and inversely proportional to square of distance between them.

$$\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$$

Characteristics of Gravitational Force

- It is alway attractive
- It is independent of the medium
- It is a conservative and central force
- It has infinite range

Superposition Principle

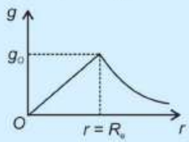
The Gravitational force on a point mass m_1 is the vector sum of the gravitational forces exerted by m_2, m_3 ....

i.e. $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...$

Due to Depth (d)

The value of g decreases with depth

$$g_d = g\left(1 - \frac{d}{R_e}\right)$$



3 ACCELERATION DUE TO GRAVITY

- For a body falling freely under gravity, the acceleration of body is called acceleration due to gravity
- $g = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi G\rho R_e$

Where G = Gravitational constant

$\rho \to$ Average density of earth

$M_e \to$ Mass of earth

$R_e \to$ Radius of earth

5 GRAVITATIONAL POTENTIAL ENERGY

- The work done in bringing a body from infinity to a point in the gravitational field is gravitational potential energy

For two point mass system

$$U = -\frac{Gm_1m_2}{r}$$

Gravitational Potential due to a point mass

It is the work done in bringing a unit mass from infinity to a point in the gravitational field. $V = -\frac{Gm}{r}$

6 ESCAPE SPEED

- The minium speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e} = \left(\sqrt{\frac{8\pi G\rho}{3}}\right)R_e$$

It is independent of angle of projection.

- Escape velocity from moon is about 5 times smaller than earth.

7 EARTH'S SATELLITE

Orbital Speed of Satellite

- The speed required to put satellite into a given circular orbit

$$v_0 = \sqrt{\frac{GM_e}{R_e+h}} = R_e\sqrt{\frac{g}{R_e+h}}$$

- For satellite very close to earth orbital speed

$$v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e} = \frac{v_e}{\sqrt{2}}$$

Time Period of Satellite

- $T = \frac{2\pi}{\sqrt{GM_e}}(R_e+h)^{3/2} = \frac{2\pi}{R_e}\sqrt{\frac{(R_e+h)^3}{g}}$
- For satellite very close to earth's surface

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}$$

Energy of Satellite

- Kinetic energy $K = \frac{GM_em}{2(R_e+h)}$
- Potential energy $U = -\frac{GM_em}{(R_e+h)}$
- Total energy $(E) = K + U$

$$= -\frac{GM_em}{2(R_e+h)}$$

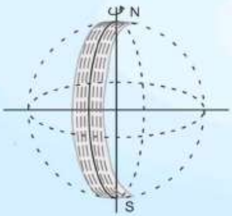
- Binding energy $(BE) = -E$

$$= +\frac{GM_em}{2(R_e+h)}$$

8 TYPES OF SATELLITES

Polar Satellite

- Revolves in polar orbit around the earth
- Height is approximately 500 to 800 km
- Time period is nearly 100 min
- Used in military spying, weather forecasting, meteorology etc.



Geostationary Satellite

- Time period is 24 h.
- Height is approximately 35800 km.
- Have same angular speed and sense of rotation as of earth
- Used for satellite communication, GPS
- INSAT is group of Geostationary satellites sent up by India.

9 WEIGHTLESSNESS

An Astronaut experiences weight- lessness in a space satellite. This is not because the gravitational force is small at that location in space. It is because both the astronaut and every part of satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position.

Mechanical Properties of Solids

THE PHYSICS SPECIALIST

9 Chapter

1 ELASTICITY AND PLASTICITY

- **Elasticity :** Property of a body to regain its original shape and size, on removing the deforming force
- **Plasticity:** The inability of a body to regain its original size and shape on the removal of the deforming forces

2 STRESS AND STRAIN

- Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$ unit : $N\,m^{-2}$

(a) **Longitudinal stress :**

Tensile stress : When a cylinder is stretched by two equal forces normal to its cross-sectional area the restoring force per unit area is called Tensile stress.

Compressive stress : If the cylinder is compressed under the action of applied forces, the restoring force per unit area is called compressive stress.

(b) **Tagential stress (or shear stress) :** The restoring force per unit area developed due to applied tangential force is called tangential or shearing stress.

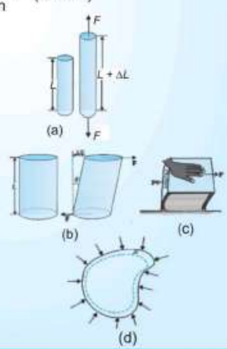
(c) **Hydraulic stress :** It is the restoring force per unit area. When a body under high pressure is compressed uniformly on all sides, the magnitude is equal to hydraulic pressure.

- Strain = $\frac{\text{Change in dimension}}{\text{Original dimension}}$ (No unit)

(a) Longitudinal strain = $\frac{\Delta L}{L}$

(b) Shear strain = $\frac{\Delta x}{L} = \tan\theta$

(c) Volume strain = $\frac{\Delta V}{V}$



3 HOOKE'S LAW

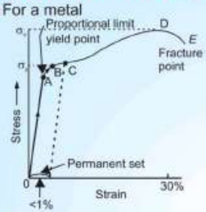
Stress ∝ strain

Stress = k strain

k = modulus of elasticity

4 STRESS - STRAIN CURVE

- For a metal



O to A : linear curve(Hooke's Law)

A : Proportional limit

B : Yield point(elastic limit) → Corresponding stress is yield strength(σ_y)

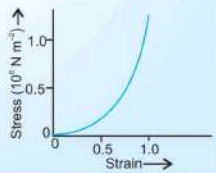
D : Ultimate tensile strength (σ_u)

E : Fracture point

- Material is brittle if D and E are close and ductile if D and E are far apart

- For an elastomer

Very large elastic region, even if material does not obey Hooke's law and there is no well defined plastic region.



5 VARIOUS OF MODULUS OF ELASTICITY

(a) Young's Modulus

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{FL}{A\Delta L}$$

(b) Bulk Modulus

$$B = \frac{\text{hydraulic stress}}{\text{volume strain}}$$

$$= -\frac{p}{(\Delta v/v)}$$

Compressibility $k = \frac{1}{B}$

(c) Shear modulus or modulus of rigidity

$$G = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{FL}{A\Delta x}$$

For most materials, $G = \frac{Y}{3}$

6 POISSON'S RATIO

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(\Delta d/d)}{(\Delta L/L)}$$

Δd : Contraction in diameter of stretched wire.

7 ELASTIC POTENTIAL ENERGY (IN A STRETCHED WIRE)

$$U = \frac{1}{2}\ YA \times \frac{\Delta L^2}{L} = \frac{1}{2}F\Delta L$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

Elastic potential energy per unit volume

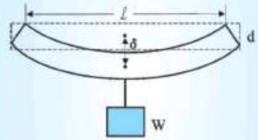
$$u = \frac{1}{2}\ \text{stress} \times \text{strain} = \frac{1}{2}\sigma\varepsilon$$

8 APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

- Minimum area of cross - section of wire of crane

$$A = \frac{Mg}{\sigma_y}$$

- Designing beams for bridges



$$\delta = \frac{Wl^3}{(4bd^3Y)}$$

$\delta \propto d^{-3}$

So I shaped beam is preferred

- Maximum height of a mountain

$h = \frac{E}{\rho g}$, E is elastic limit

Mechanical Properties of Fluids

Chapter 10

① PRESSURE

- Average pressure is defined as the normal force acting per unit area

$P_{av} = \frac{F}{A}$

$P = \lim_{\Delta A \to 0}\left(\frac{\Delta F}{\Delta A}\right)$, It is a scalar quantity.

② VARIATION OF PRESSURE WITH DEPTH

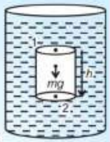
Pressure difference

$P_2 - P_1 = \rho g h$

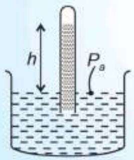
If point 1 is at free surface of liquid

then $P_2 = P_a + \rho g h$

$P - P_a = \rho g h$ (called gauge pressure)

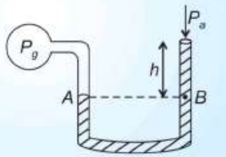


- **Instruments used to measure pressure**



Mercury barometer used to measure atmospheric pressure

$P_a = \rho g h$



The open tube manometer, used to measure pressure of gas

$P_g = P_a + \rho g h$

③ PASCAL'S LAW

- When ever an external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

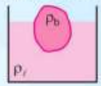
Devices based on Pascal's law

(i) Hydraulic lift (ii) Hydraulic brakes

④ ARCHIMEDES PRINCIPLE

- Loss of weight of a body submerged (partially or completely) in a fluid is equal to the weight of the fluid displaced.

Weight of fluid displaced = $\rho_\ell V_s g = F_B$



If $\rho_b < \rho_\ell$; then body will float

If $\rho_b = \rho_\ell$; body will just float with fully submerged

If $\rho_b > \rho_\ell$; then body will sink

- **Law of Floatation**

For Floating object $\frac{V_s}{V_b} = \frac{\rho_b}{\rho_\ell}$

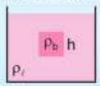
Fraction of vol. submerged = ratio of density of body and fluid

- **Buoyant Force**

Buoyant force is equal to weight of the fluid displaced.

Buoyant force depends on g_{eff}

Buoyant force acts opposite to g_{eff}



⑤ STREAMLINE FLOW

- The flow is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time. The path taken by fluid particle under steady flow called streamline.
- **Equation of continuity:** In stream line flow, mass of liquid coming out equals to the mass of liquid flowing in

$A_1 v_1 = A_2 v_2$

It is based on conservation of mass.

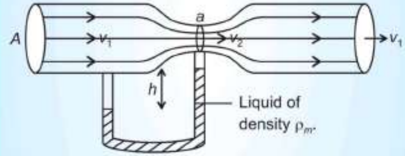
⑥ BERNOULLI'S EQUATION

- It states that for a steady flow of an ideal fluid, the sum of pressure energy per unit volume (*P*), kinetic energy per unit volume and potential energy per unit volume remains constant.

$P + 1/2\,\rho v^2 + \rho g h = \text{constant}$

- Phenomenons associated: Heart attack, magnus effect and aerofoil (lift of aircraft)
- **Venturi-meter:** It is a device used to measure the flow speed of incompressible fluid.

$v_1 = \sqrt{\frac{2\rho_m g h}{\rho}}\left[\left(\frac{A}{a}\right)^2 - 1\right]^{-1/2}$

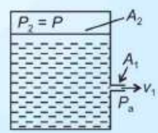


- **Speed of efflux: Torricelli's Law**

$A_1 v_1 = A_2 v_2$, if $v_2 << v_1$

$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$

- When $P >> P_a$ and $2gh$ may be ignored.
- On the other hand tank is open to atmosphere, then $P = P_a$



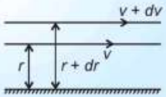
$v_1 = \sqrt{2gh}$

7 VISCOSITY

- The property of fluid due to which it opposes relative motion between its different layers in steady flow called viscosity.
- Tangential force between the layer

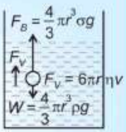
  $F = -\eta A \frac{dv}{dr}$

  η = coefficient of viscosity



- $\eta = \frac{\text{Shearing stress}}{\text{Shear strain rate}}$
- SI unit is poiseiulle (ρ_l).

8 STOKE'S LAW

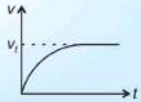


- The viscous force acting on a spherical body of radius *a*. $F = -6\pi\eta av$
- **Terminal velocity:**

  $v_t = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

  ρ = density of sphere material

  σ = density of fluid
- **Variation of Velocity with Time**



9 REYNOLDS NUMBER (R_e):

$R_e = \frac{\text{Inertial force}}{\text{Viscous force}}$ (dimensionless)

$R_e = \frac{\rho vd}{\eta}$

Where v = velocity of liquid

ρ = density of liquid

- The flow is turbulent for $R_e > 2000$.
- Flow is unsteady for R_e between 1000 and 2000.
- Flow is streamline for R_e less than 1000.
- Critical Reynold number is one at which turbulence sets.
- Reynold number helps study nature of fluid flow.
- Turbulence dissipates kinetic energy in the form of heat.

10 SURFACE TENSION

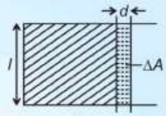
- **Surface:** It is the thickness of few molecular size.
- **Surface Tension:** The property of liquid by which the free surface of liquid at rest tends to have minimum surface area called surface tension.

  Surface tension can be defined as the force per unit length on imaginary line drawn at the surface of liquid

  $S = \frac{F}{l}$
- Surface tension of a liquid falls with temperature.
- **Surface energy:** Molecules on the surface of liquid have some extra potential energy in comparison to molecules in the interior. A liquid thus tends to have minimum surface area.

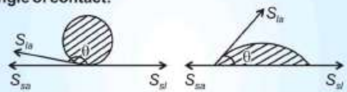
11 SURFACE ENERGY AND SURFACE TENSION

- Work done in increasing surface area.



  $W = S(\Delta A_{eff})$
  - Thin film, liquid bubble have two surfaces so,

    $\Delta A_{eff} = 2\Delta A_{Geo}$.
- **Angle of contact:**

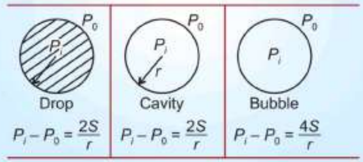


- At the point of contact, the angle between tangent planes drawn at the surface of liquid and at surface of solid inside liquid called angle of contact.
  - If $\theta < 90°$ → Surface will be concave, liquid stick to solid and rise in capillary.
  - If $\theta > 90°$ → Surface will be convex, liquid does not stick to solid and fall in capillary.
  - If $\theta = 90°$ → Surface will be plane, liquid does not stick to solid neither rise nor fall in capillary.
  - Water forms droplets over a lotus leaf while spreads over a clean plastic plate.

12 DROPS AND BUBBLES

- $P_{inside} > P_{outside}$ (For liquid-gas interface, the convex side has lower pressure than on concave side.)
  - Liquid drop, air bubble in water have one surfaces so,

    $\Delta A_{eff} = \Delta A_{Geo}$

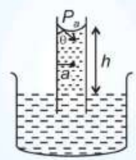


13 CAPILLARITY

- When a capillary tube is dipped in any liquid then liquid either rise or fall inside the capillary tube.
- Height of liquid column rise or fall inside a capillary tube is

  $h = \frac{2S\cos\theta}{a\rho g}$

  $h \propto \frac{1}{a}$



- In a tube of insufficient length, liquid will rise to the top of capillary, increase radius of curvature and stay there. Never comes out in the form of fountain.
- This is consequence of pressure difference across a curved liquid air interface a well known effect that water rises up in narrow tube inspite of gravity.

Thermal Properties of Matter

THE PHYSICS SPECIALIST

11 Chapter

1 TEMPERATURE

- Temperature is a relative measure of hotness or coldness.
- Heat transfer takes place between system and surrounding medium until they are at same temperature.
- Measure of temperature is obtained using a thermometer.
- Some properties of material change with temperature to be used as basis of constructing thermometer.
- For standard scale a fixed reference point is taken.
- A relationship for conversion between Fahrenheit and Celsius temperature scale is

$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

- A temperature –273.15°C is designated as absolute zero. This is foundation of Kelvin temperature scale.
- Size of unit of Kelvin and Celsius temperature scales is the same. Relation between scales is

$$T_K = t_C + 273.15$$

2 HEAT

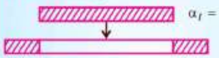
A form of energy transferred between two or more systems by virtue of temperature difference.

Thermal Expansion

- A change in temperature of a body causes change in its dimensions.

Three types of expansion

1. Linear Expansion



$$\alpha_l = \frac{\Delta l}{l} \times \frac{1}{\Delta T}$$

2. Area Expansion

$$\alpha_a = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

$\alpha_a = 2\alpha_l$

For anisotropic solid $\alpha_a = \alpha_{l_1} + \alpha_{l_2}$

3. Volume Expansion

$$\alpha_V = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

- α_V is constant only at high temperature
- Pyrex glass and invar has low α_V.
- Alcohol has high volume expansion coefficient than mercury.
- $\alpha_V = \frac{1}{T}$ for ideal gases

  $(\alpha_V)_{gases} > (\alpha_V)_{liquid} > (\alpha_V)_{solids}$
- When a solid rod has its ends rigidly fixed, it results in thermal stress in material which is proportional to temperature change.

$$\text{Thermal Stress} = Y \cdot \alpha_l \Delta T$$

3 CALORIMETRY

- Heat lost by a part at higher temperature is equal to heat gained by the part at lower temperature.
- Calorimetry means measurement of heat.
- A device in which heat measurement can be done is called a calorimeter.

4 HEAT CAPACITY

The change in temperature of a substance, when a given quantity of heat is absorbed or rejected is characterised by a quantity called heat capacity.

$$S = \frac{\Delta Q}{\Delta T}$$

Specific heat capacity

This is unique value of heat absorbed or given off, to change unit mass of it by one unit temperature change.

$$s = \frac{S}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

Molar specific heat

If the amount of substance is specified in terms of moles we define heat capacity per mole

$$C = \frac{S}{\mu} = \frac{1}{\mu}\left(\frac{\Delta Q}{\Delta T}\right) \quad \text{J mol}^{-1}\,\text{K}^{-1}$$

For gases two molar specific heat capacities

- Molar specific heat capacity at constant pressure C_P
- Molar specific heat capacity at constant volume C_V

$C_P - C_V = R$ (for ideal gases)

5 CHANGE OF STATE

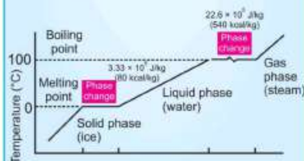
- Change of state from solid to liquid is called melting or fusion.
- Change of state from liquid to vapour is called vaporisation
- The temperature at which the liquid and vapour states of substance coexist is called its Boiling point.
- Boiling point increases with increase in pressure and vice versa.
- The change from solid state to vapour state without passing through the liquid state is called sublimation and substance is said to sublime.

Latent heat

- Amount of heat transferred during change of state of substance is called its latent heat.

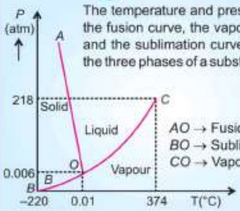
$$L = \frac{\Delta Q}{M} \quad \text{J kg}^{-1}$$

- L depends on pressure.
- Solid-liquid state change → Latent heat of fusion (L_f)
- Liquid-gas state change → Latent heat of vaporisation (L_v)



6 TRIPLE POINT

The temperature and pressure at which the fusion curve, the vaporisation curve and the sublimation curve meet and all the three phases of a substance coexist



AO → Fusion curve
BO → Sublimation curve
CO → Vaporisation curve

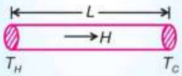
Pressure-temperature phase diagrams for water

HEAT TRANSFER MODES

7 CONDUCTION

- It is a mechanism of heat transfer between adjacent parts of a body due to temperature difference.
- At steady state, the temperature of bar, through which heat is flowing, decreases with distance, and heat starts flowing at a constant rate.
- The rate of flow of heat

$$H = KA\left(\frac{T_H - T_C}{L}\right)$$



T_H = Hot end Temperature

T_C = Cold end Temperature

L = Length of rod

A = Cross-section of rod

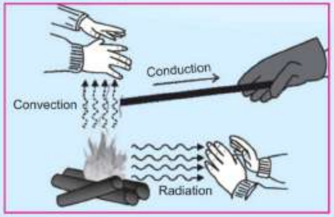
K is called thermal conductivity of material.

- Greater value of *K* for a material, more rapidly will it conduct heat. Its SI units are $Wm^{-1}K^{-1}$

8 CONVECTION

In convection mode heat transfer by actual motion of matter occurs.

- Convection occurs in fluids only.
- Two types of convections are
  1. Natural convection
  2. Forced convection
- Trade winds is example of natural convection in which gravity plays an important role.
- In forced convection material is forced to move by a pump. Human circulatory system, cooling system of automobile engine are forced convection method.



9 RADIATION

- This heat transfer mechanism needs no medium.
- Energy transferred by waves is called radiant energy.
- Heat transferred from sun to earth is by radiation.
- Radiations emitted by hot bodies are called thermal radiations.

LAWS OF RADIATIONS

10 Wien's Displacement Law

Wavelength for which radiation energy is maximum decreases with increasing temperature.

$$\lambda_m T = \text{constant}$$

Value of constant (Wien's constant) is 2.9×10^{-3} m K.

- This law is used to measure surface temperature of celestial bodies like stars, moon and sun.

11 Stefan-Boltzmann's Law

For a black body which is perfect radiator energy emitted per unit time is given as

$$H = A\sigma T^4$$

A is area, *T* is absolute temperature of body, σ is called Stefan Boltzmann's constant

$\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$

- But if a body is surrounded by surroundings at temperature T_s.

For perfect radiator net rate of heat radiated

$$H = \sigma A(T^4 - T_s^4)$$

- For body with emissivity *e* modified relation is

$$H = e\sigma A(T^4 - T_s^4) \quad 0 < e \le 1$$

12 Newton's law of cooling

cooling curve

Rate of heat loss of a body is directly proportional to difference of temperature of body and surroundings.

- This law holds for small temperature difference only.

$$-\frac{dQ}{dt} = k(T_B - T_S)$$

$$\log_e (T_2 - T_1) = -kt + C$$

Average method

$$\frac{\text{Change in temperature}}{\text{time}} = k\Delta T$$

$$\Delta T = (T_{av} - T_s)$$

13 Greenhouse effect: The absorption of infrared waves by greenhouse gases such as CO_2, methane (CH_4) nitrous dude (N_2O) chlorofluorocarbon (CF_xCl_x) and ozone (O_3). Heating of atmosphere → More energy to earth → Warmer surface. Without Greenhouse effect temperature of earth would have been −18°C

Thermodynamics

THE PHYSICS SPECIALIST

12
Chapter

1 THERMODYNAMIC EQUILIBRIUM

- Temperature of a body is related to its average internal energy, not to kinetic energy of motion of centre of mass.
- Equilibrium in thermodynamics refer to situation when macroscopic variables defining thermodynamic state of system don't depend on time.

2 ZEROTH LAW OF THERMODYNAMICS

- Two systems in thermal equilibrium with third system separately are in thermal equilibrium with each other.
- If $T_A = T_C$ and $T_B = T_C$, then $T_A = T_B$
- Thermodynamic variable whose value is equal for two systems in thermal equilibrium is called temperature.

3 HEAT, INTERNAL ENERGY AND WORK

- Heat is energy transfer arising due to temperature difference between system and surroundings.
- Internal energy is simply the sum of kinetic energies and potential energies of the molecules in the frame of reference to which centre of mass of system is at rest.
- Internal energy depends on state of the system, not how the state was achieved.
- There are two ways to change internal energy of a thermodynamic system

  (1) To do work on system

  (2) Supply heat to system

  So heat and work are two modes of altering the state of a thermodynamic system and changing internal energy.
- Heat and work in Thermodynamics are not state variables.
- U is a state variable. ΔU depends only on initial and final states and not on path taken by gas to go from one to another.
- ΔQ and ΔW will depend on path taken to go from initial to final state.
- Work done during thermodynamic process

  $$\Delta W = \int_{V_1}^{V_2} P dV$$
- Area under the $P-V$ diagram with the volume axis gives the work done in thermodynamic process.

4 SPECIFIC HEAT CAPACITY

- Molar specific heat at constant volume.

  $$C_V = \left(\frac{\Delta Q}{\Delta T}\right)_V = \left(\frac{\Delta U}{\Delta T}\right)$$
- Molar specific heat at constant pressure

  $$C_P = \left(\frac{\Delta Q}{\Delta T}\right)_P = \left(\frac{\Delta U}{\Delta T}\right)_P + P\left(\frac{\Delta V}{\Delta T}\right)_P$$

  $$PV = RT \therefore P\left(\frac{\Delta V}{\Delta T}\right)_P = R$$
- $\boxed{C_P = C_V + R}$ (MAYER'S Equation)
- $\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$

  $\boxed{C_P = \gamma \times C_V}$

5 FIRST LAW OF THERMODYNAMICS

- $\Delta Q = \Delta U + \Delta W$ (Energy conservation law)

  ΔQ = heat supplied to system by the surrounding

  ΔW = work done by the system on the surrounding

  ΔU = Change in internal energy of a the system
- Heat supplied to system goes in partly to increase internal energy and rest in work on environment.
- This is simply the general law of conservation of energy applied to any system in which energy transfer is taken into account.
- $\Delta W = P\Delta V$

$\therefore$ $\Delta Q = \Delta U + P\Delta V$

8 THERMODYNAMIC PROCESSES

- A thermodynamic process is an activity where a thermodynamic system is taken from one equilibrium state to another.
- Reversible process
- Irreversible process
- Cyclic process

6 THERMODYNAMIC STATE VARIABLES

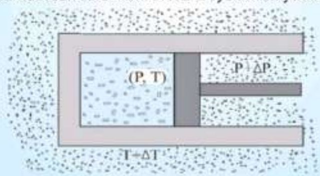
- Thermodynamic state variables describe equilibrium state of system. These state variables are not necessarily independent.
- The connection among state variables is called equation of state.
- Equilibrium state of thermodynamic system is described by state variables. The value of state variable depends on particular state not by the path used to arrive that state. Pressure, volume, temperature and mass are state variable. Heat and work are not state variables.
- For an ideal gas, equation of state is $\boxed{PV = \mu RT}$
- Thermodynamic state variables are of two types

  (1) Extensive

  (2) Intensive
- Extensive variables indicates size of system.
- Internal energy, volume and mass are extensive variables. But pressure, temperature and density are intensive variables.

7 REVERSIBLE AND IRREVERSIBLE PROCESS

- Spontaneous processes in nature are irreversible.
- A process is reversible if the process can be turned back such that both the system and surrounding return to their original states with no any other change anywhere else in universe.
- A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.
- A quasi-static process is an infinitely slow process such that system remains in thermal and mechanical equilibrium with surroundings throughout. In this process pressure and temperature of the environment can differ from those of system only infinitesimally.



No accelerated motion of Piston

9 ISOTHERMAL PROCESS

- For isothermal process Temperature during the process should be constant
  $PV = \text{constant}$
- So pressure of given mass of a gas varies inversely as its volume.
- **Work done in isothermal process.**
  If a system of ideal gas at temperature T goes from (P_1, V_1) to (P_2, V_2) equilibrium state, then work done
  $$W = \mu RT \ln\left(\frac{V_2}{V_1}\right) = \mu RT \ln\left(\frac{P_1}{P_2}\right)$$
- Here $\Delta T = 0$ $\quad \therefore \Delta U = 0$
  $$\Delta Q = \Delta W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$$

10 ADIABATIC PROCESS

- In adiabatic process heat interaction between system and surrounding is zero. *i.e.* $\Delta Q = 0$
- $PV^{\gamma} = \text{constant}$
  Where γ = ratio of molar specific heats at constant pressure and at constant volume.
- System is insulated from surroundings and heat absorbed or released is zero.
- Work done by gas results in decrease in its internal energy.
- If system change from (P_1, V_1, T_1) to (P_2, V_2, T_2)
  $$\Delta W = \frac{\mu R(T_1 - T_2)}{\gamma - 1} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} \text{ where } \gamma = C_P/C_V$$
- If work done by gas $(W > 0)$, then, $T_2 < T_1$

11 ISOBARIC PROCESS

- For isobaric process pressure during the process should be constant
  $\frac{V}{T} = \text{constant}$
- Work done in isobaric process
  $W = P(V_2 - V_1) = \mu R(T_2 - T_1)$
- Heat partly to do absorbed goes partly to increase internal energy and mechanical work.
  $\Delta Q = \Delta U + \Delta W$
  $\Delta U = \mu C_V \Delta T$, $\Delta Q = \mu C_P \Delta T$ and $\Delta W = \mu R \Delta T$
  $$\frac{\Delta W}{\Delta Q} = \frac{R}{C_P} = \frac{\gamma - 1}{\gamma} \text{ and } \frac{\Delta U}{\Delta Q} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

12 ISOCHORIC PROCESS

- For isochoric process volume during the process should be constant
  $\frac{P}{T} = \text{constant}$
- Work done in isochoric process, $\Delta W = P\Delta V = 0$
- $\Delta Q = \Delta U + \Delta W$
  $\Delta Q = \Delta U$
- Heat absorbed by gas goes entirely to change its internal energy and its temperature.
- Change in internal energy is determined by specific heat at constant volume and temperature change.

13 CYCLIC PROCESS

- In any cyclic process system returns to initial state, $\Delta U = 0$
- Hence total heat absorbed equals the work done by the system, $\Delta Q = \Delta W$

15 REFRIGERATOR

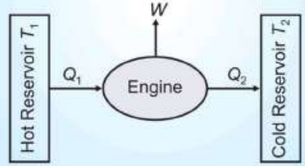
- A refrigerator is the reverse of a heat engine. Working substance extracts heat from cold reservoir, some external work is done on system and heat is released to reservoir at high temperature.
- Coefficient of performance of refrigerator $= \frac{\text{heat extracted}}{\text{work input}}$
  $$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{(T_1 - T_2)} = \frac{1-\eta}{\eta}$$
- Coefficient of performance for heat pump is
  $$\beta = \frac{Q_1}{W} = \frac{T_1}{T_1 - T_2} = \frac{1}{\eta}$$

16 SECOND LAW OF THERMODYNAMICS

- **Kelvin-Planck statement :** No process is possible whose sole result is absorption of heat from a reservoir and complete conversion of heat into work.
- **Clausius statement :** No process is possible whose sole result is transfer of heat from cold reservoir to hotter object.
- Two statements are completely equivalent.
- It shows that efficiency of a heat engine can never be unity so heat released to cold reservoir can never be made zero.
- Kelvin Planck and Clausius deny the perfect heat engine and refrigerator.

14 HEAT ENGINE

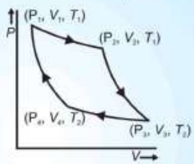
- Heat engine is a device in which a system undergoes a cyclic process resulting in conversion of heat in to the sink.
- Efficiency of the engine is
  $$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$
  Q_1 = heat absorbed from source
  Q_2 = heat released to sink
  η = efficiency of heat engine



- Heat engine based on idealised reversible processes achieve the highest possible efficiency.

17 CARNOT ENGINE

- Carnot engine is a reversible engine operating between two temperatures T_1 and T_2. Carnot cycle consists of two isothermal and two adiabatic processes. Its efficiency is
  $$\eta = 1 - \frac{T_2}{T_1}$$



- Engine efficiency of Carnot engine does not depend on nature of working substance.
  Carnot Theorem: Any other engine working between temperature T_1 and T_2 can not have efficiency more than that of Carnot engine. The Carnot engine's efficiency is independent of nature of working substance.
  In Carnot cycle
  $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ is universal relation and this relation can be used to design universal thermodynamic scale.

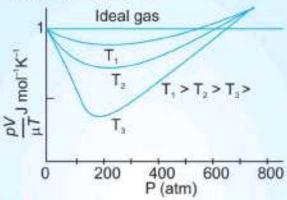
Kinetic Theory

THE PHYSICS SPECIALIST

13 Chapter

① LAWS ASSOCIATED WITH KTG

- An ideal gas is only theoretical model of a gas. No real gas is truly ideal. Without interactions gas behaves like an ideal gas. At low pressure or high temperature, molecules are far apart and molecular interaction is negligible.



- **Boyle's Law :** Pressure of a given ideal gas is inversely proportional to its volume if temperature is kept constant.
- **Charle's Law :** Volume of given ideal gas is directly proportional to its temperature in kelvin if pressure is kept constant.
- **Gay Lussac's Law :** Pressure of an ideal gas is directly proportional to its absolute temperature if volume is kept constant.
- **Avogadro's Law :** Equal volume of all the gases under similar conditions of temperature and pressure contain equal number of molecules.

$$\frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2} = K_B$$

Ideal gas equation connecting the variables is

$$\boxed{PV = \mu RT = K_BNT} \quad P = \frac{\rho RT}{M_0}$$

- **Dalton's Law of Partial Pressure :** Total pressure of a mixture of non-reactive gases is the sum of their partial pressures.

② AVERAGE PRESSURE OF GAS

$$\boxed{P = \frac{1}{3} n\, m\bar{v}^2 \text{ and } PV = \frac{1}{3} nV\, m\bar{v}^2}$$

$n \rightarrow$ Number density

$m \rightarrow$ Mass of molecule

$\bar{v}^2 \rightarrow$ Mean of squared speed

③ KINETIC INTERPRETATION OF TEMPERATURE

- **Average kinetic energy of molecule**

$$= \frac{1}{2} m\bar{v}^2 = \frac{3}{2} K_BT$$

$$\bar{V}_{RMS} = (\bar{V}^2)^{\frac{1}{2}}$$

$$= \sqrt{\frac{3K_BT}{m}}$$

- In a mixture of gases at a given temperature, heavier molecule has lower average speed.
- Translational kinetic energy of gas

$$\boxed{E = \frac{3}{2} K_BNT} \text{ and } \boxed{PV = \frac{2}{3} E}, \frac{E}{N} = \frac{1}{2} m\bar{v}^2 = \frac{3}{2} K_BT$$

- Average KE is proportional to temperature.
- R.M.S. speed of gas molecule,

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

- Most probable speed of molecule

$$= \sqrt{\frac{2RT}{M}}$$

- Average speed of gas molecule

$$= \sqrt{\frac{8RT}{\pi M}}$$

- This concept of maxwell energy distribution predict specific heat of gases theoretically.

LAW OF EQUIPARTITION OF ENERGY

- KTG is consistent with ideal gas equation.
- For a system in equilibrium at absolute temperature T, total energy is distributed equally in different modes of absorptions. Energy of each mode is equal to $1/2\ K_BT$.
- Each translational and rotational degree of freedom corresponds to one energy mode of absorption.

④ SPECIFIC HEAT CAPACITIES

- Specific heat capacity for solids = $3R$
- Specific heat capacity of water = $9R$
- C_v (monoatomic gas) = $\frac{3}{2}R$
- $\gamma = 1 + \frac{2}{f}$
- $\gamma = \frac{5}{3}$ (monoatomic)
- $\gamma = \frac{7}{5}$ (rigid diatomic)
- Polyatomic gases in general a polyatomic molecules has 3 translational, 3 rotational degree of freedom and a certain number (f) of vibrational modes. Then for one mole of gas

$$U = \left[\frac{3}{2} K_BT + \frac{3}{2} K_BT + fK_BT\right] N_A$$

$$C_V = (3 + f)R \qquad \gamma = \frac{(4+f)}{(3+f)}$$

$$C_P = (4 + f)R$$

- Each vibrational frequency has two modes of energy with corresponding energy equal to K_BT.
- Molecules of a monatomic gas have only translational degree of freedom.
- Molecules like CO even at moderate temperature has mode of vibration.
- Diatomic molecule, like a dumbell, has five degree of freedom.
- Polyatomic molecule has 3 transitional, 3 rotational and a degree of a certain number of vibrational modes.

⑤ MEAN FREE PATH

- Molecules of gas have rather large speeds of the order of speed of sound.
- Molecules of gas undergo collisions and their paths keep getting deflected.
- Average distance a molecule can travel without collision is called mean free path.
- Mean free path of gas molecule is related to number of molecules per unit volume and size of gas molecule.

$$\boxed{\lambda = \frac{1}{\sqrt{2}\pi nd^2}} = \boxed{\lambda = \frac{K_BT}{\sqrt{2}\pi Pd^2}}$$

n : number density; d : diameter of molecules

- Mean free path in gases is of order of thousands of angstrom.

P : Pressure of gas; T : Absolute temperature

K_B : Boltzmann's Constant

Oscillations

Chapter 14

1 SPECIAL TYPES OF MOTIONS

- **Periodic Motion**

  A motion which repeats itself after regular intervals of time, (T) is periodic. Examples:

  - Motion a particle in circle with constant speed
  - Skipping
  - Spring block system
  - Simple pendulum
  - Motion of Earth around sun
  - Motion of needle of sewing machine
  - A boat tossing up and down in a lake
  - Piston of engine going back and forth can be periodic

- **Oscillatory Motion**

  Special type of periodic motion in which a particle moves to and fro about a fixed point. The force acting on the particle in a direction directed towards equilibrium position is called **restoring force**.

  - Every oscillatory motion is periodic but every periodic motion may not be oscillatory.
  - Back and Forth motion can be oscillatory or vibratory. When oscillations frequency is small we call it oscillatory, at high frequency we call it vibratory.

  Oscillations can be

  A. **Free oscillations**
  - When a system oscillates with its natural frequency the oscillations are called free oscillations.

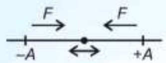
  B. **Damped oscillations**
  - If some external resisting force appears opposing restoring force, oscillatory amplitude starts decreasing with time.

  C. **Forced oscillations**
  - Forced oscillations are those in which damping is not allowed by applying an external time varying force, which compensates the effect of damping force acting on it.

2 SIMPLE HARMONIC MOTION

- Simple harmonic motion is an example of periodic oscillatory motion.
- Special type of oscillatory motion which satisfies following conditions.
  A. Oscillatory amplitude of particle is small.
  B. During oscillation, acceleration towards mean position, due to net restoring force, is directly proportion to displacement from mean position.
- Force displacement relation in S.H.M.
  $F = -ky$, where K is force constant (Force law in S.H.M.), y is displacement from mean position.
- Acceleration of particle

  $a = \frac{F}{m} = -\left(\frac{K}{m}\right)y = -\omega^2 y$



  ∴ Acceleration and displacement are antiparallel

  $\frac{d^2y}{dt^2} + \omega^2 y = 0$, here $\omega = \sqrt{\frac{K}{m}}$ (Angular frequency)

  m is mass oscillating, K is force constant.

- **General equation for displacement in S.H.M.**

  $y = A\sin(\omega t + \phi)$ or $y = A\cos(\omega t \pm \phi)$

  $\omega = \frac{2\pi}{T} = 2\pi n$ is angular frequency and $(\omega t + \phi)$ is called phase, a time varying quantity.

  Here ϕ is called **epoch** or initial phase.

  A. If particle at $t = 0$ is at equilibrium position. $(y = 0)$

  $y = A\sin\omega t$

  B. If particle at $t = 0$ is at extreme right position $(y = A)$

  $y = A\cos\omega t$

- **Velocity of particle in SHM.**

  $v_P = \frac{dy}{dt} = \omega A\cos(\omega t \pm \phi)$

  If at $t = 0$ particle is at origin.

  $v_P = \omega A\cos\omega t = \omega\sqrt{A^2 - y^2}$

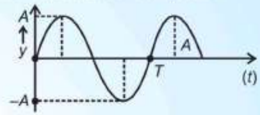
- **Acceleration of particle in SHM**

  $a_p = -\omega^2 A\sin\omega t$, at $t = 0$ particle is at mean position.

- Velocity displacement graph will be an ellipse $(\omega \neq 1)$ or a circle $(\omega = 1\text{ rad s}^{-1})$.
- The maximum velocity of particle executing SHM will be at mean position and at extremes speed becomes minimum (zero).
- Different graphs for a particle executing SHM

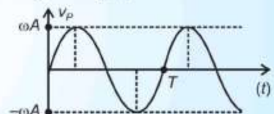
  (A) **Displacement - time graph**

  If at $t = 0$ particle is at mean position



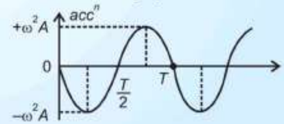
  $y = A\sin(\omega t)$

  (B) **Velocity - time graph**



  $v = A\omega\cos(\omega t)$

  (C) **Acceleration time graph**

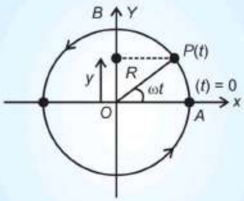


  $a = -\omega^2 A\sin(\omega t)$

  - Velocity leads displacement by a phase of $(\pi/2)$ rad.
  - Acceleration leads velocity a phase of $\pi/2$ rad.

(3) SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

- Projection of uniform circular motion on a diameter of the circle follows simple harmonic motion.



Particle position
$y = R \sin \omega t$ is SHM.

This is an equation of S.H.M. for particle displacement at any time.

(4) SPECIAL PARAMETERS IN SHM

- Since particles speed is not constant ; from mean position to half of amplitude it takes half of time than to move from half of amplitude to extreme position.
- Minimum velocity in S.H.M. is $v_{min} = 0$ at extremes and maximum velocity at equilibrium position. $v_{max} = \omega A$
- Maximum acceleration of particle is at extreme positions $a_{max} = \omega^2 A$ and minimum (zero) is at equilibrium.
- Maximum force on particle is at extreme positions and zero at mean, in between it varies linearly always directed towards equilibrium.

  $F_{max} = m\omega^2 A$ and F_{min} = zero

(5) MECHANICAL ENERGY IN SIMPLE HARMONIC OSCILLATOR

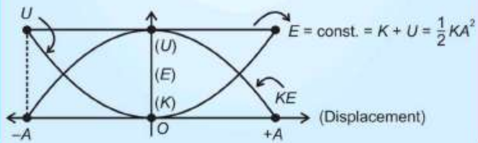
- Potential energy in SHM $U = \frac{1}{2}ky^2 + U_0$

  U_0 is generally taken zero at equilibrium.

  $F_{int} = -\frac{dU}{dy}$; instantaneous force on particle.
- Maximum potential energy occurs at extreme positions and minimum at mean position.
- Graph of potential energy versus displacement of particle will be parabolic, symmetric about y-axis.
- Kinetic energy of particle in S.H.M. varies directly as square of its velocity at any location.

  $KE = \frac{1}{2}(m\omega^2)(A^2 - y^2) = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$
- Kinetic energy can not be negative. Potential energy increases at expense of KE and vice versa.
- Kinetic energy will be maximum at mean position and zero at extreme position.
- Total mechanical energy is independent of time.
- Potential energy and kinetic energy peaks twice during every period. Element of springiness stores potential energy and element of inertia stores its kinetic energy.
- Graph of kinetic energy versus position of particle will be an inverted parabola.
- In absence of damping ; total mechanical energy of harmonic oscillator will remain constant.

  $E = k_{max} = U_{max} = \frac{1}{2}m\omega^2 A^2$
- Potential energy and kinetic energy is periodic with period $\frac{T}{2}$.
- The graphs for energy versus position are



(6) OSCILLATIONS DUE TO A SPRING

(1) **Oscillations of a spring block system**

(Linear S.H.M.)

Force law, $F = -kx$

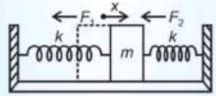
$F = -ma = -m\omega^2 x$

$\therefore \; k = m\omega^2$ or $\omega = \sqrt{\frac{k}{m}}$

Where k spring constant of spring and m is mass of block executing SHM.

(2) **For two Identical Springs**

This is also linear harmonic oscillator



When displaced right, restoring forces towards left

$F_1 = -kx, F_2 = -kx, F = F_1 + F_2$

$F = -2kx$

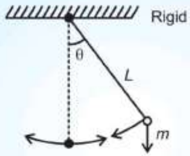
Since force acting on mass is proportional to displacement and directed towards mean position. It is SHM. The period of oscillation is

$T = 2\pi\sqrt{\frac{m}{2k}}$

7 SIMPLE PENDULUM

Simple Pendulum

- By attaching a small mass to an inextensible string, a simple pendulum can be made.
- The mass executes SHM for small displacements only.



$T = 2\pi\sqrt{\frac{I}{mgL}}$ Also here $I = mL^2$, about rigid support point.

$T = 2\pi\sqrt{\frac{L}{g}}$

- The time period of a simple pendulum depends on its length and acceleration due to gravity but is independent of its mass and amplitude.
- Time period of a clock pendulum which ticks every second is 2s and its length is approximately 1 metre.

9 FORCED OSCILLATIONS AND RESONANCE

- An external agency can maintain motion by resisting damping forces. These are called driven or forced oscillations. An external force which is periodic is applied to damped oscillator. Equation of oscillations of mass is $\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \cos\omega_d t$, and after natural oscillation, die out eqn. is

  $y = A\cos(\omega_d t + \phi)$ and A depends on ω_d and ω.

- If ω_d is close to ω then $A = \frac{F_0}{\omega_d b}$
- The phenomenon of increase of amplitude when driving frequency is close to natural frequency of the oscillators is called resonance.

8 DAMPED SIMPLE HARMONIC MOTION

- A viscous surroundings will apply force on simple pendulum or a spring pendulum and system will ultimately come to rest.
- The damping force depends on nature of surrounding medium. When damping is high, energy is quickly dissipated. This force is generally proportion to velocity of oscillator.

  $\vec{F}_d \propto \vec{v} \Rightarrow \vec{F} = -b\vec{v}$

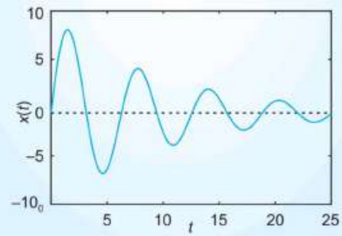
- Net force $F = -ky - bv$ ($b \rightarrow$ damping factor)

  $ma = -ky - bv$

$\frac{md^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$ is damped equation, whose solution is given by

$y = Ae^{-\frac{bt}{2m}}\cos(\omega' t + \phi)$ for displacement of oscillator.

Where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$



- Small damping means $\frac{b}{\sqrt{km}} << 1$
- $\left[E = \frac{1}{2}kA^2 e^{-bt/m}\right]$ energy eqn.

Waves

15 Chapter

WAVE

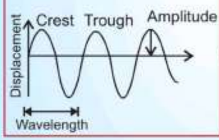
It is a disturbance produced, which transfer energy and momentum without transfer of matter.

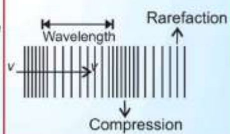
1 TYPES OF WAVES

- **Electromagnetic Wave** – wave propagates in the form of time varying electric and magnetic fields. It require no medium.
- **Matter waves** – wave associated with the particles having momentum.
- **Mechanical waves** – The waves which require a material medium for their propagation.

MECHANICAL WAVES

Transverse waves	Longitudinal waves
The individual particle of medium vibrate perpendicular to direction of propagation.	The individual particle of medium vibrate parallel to direction of propagation





- Trasverse waves are possible in solids like strings (under tension), due to shear modulus.
- Longitudinal waves, involve compressive stress, i.e. (Bulk modulus), so is possible in both solids and fluids
- Waves on the surface of water are of two kinds capillary waves and gravity waves

2 DISPLACEMENT RELATION IN A PROGRESSIVE WAVES

- $y(x, t) = a \sin(kx - \omega t + \phi)$

  a = amplitude of wave

  is linear combination of sine and cosine function

  $y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$

  Amplitude of resultant wave, $a = \sqrt{A^2 + B^2}$

  $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

  Speed of wave $v = \frac{\omega}{k} = \nu\lambda$

  $k = \frac{2\pi}{\lambda}$ called angular wave number)

  $(kx - \omega t + \phi)$ = Phase of wave

- **Speed of a Transverse Wave on a Stretched String**

  $v = \sqrt{\frac{T}{\mu}}$

  Here

  T = tension in string (in newton)

  $\mu = \frac{m}{l}$ (mass per unit length of string)

Speed of a Longitudinal wave

- Speed of longitudinal wave in a solid bar $v = \sqrt{\frac{Y}{\rho}}$

  where Y = Young's modulus of material of bar

  ρ = Density of material of bar

- Speed of longitudinal wave in gases

  According to Newton, $v = \sqrt{\frac{P}{\rho}}$ (Isothermal)

  According to Laplace, $v = \sqrt{\frac{\gamma P}{\rho}}$ (Adiabatic)

3 PRINCIPLE OF SUPERPOSITION OF WAVES

- If $y_1(x, t)$ and $y_2(x, t)$ be the displacement due to two wave disturbances in the medium and the waves arrive in a region simultaneously and overlap, the net displacement $y(x, t)$ is given by

  $y(x, t) = y_1(x, t) + y_2(x, t)$

  Similarly, resultant waveform

  $y = \sum_{i=1}^{n} f_i (x - vt)$

  In the phenomenon of **interference** of two waves

  $y_1(x, t) = a \sin(kx - \omega t)$

  and $y_2(x, t) = a \sin(kx - \omega t - \phi)$

  The net displacement

  $y(x, t) = 2a \cos\frac{\phi}{2} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$

  So, amplitude is a function of phase difference

  $A(\phi) = 2a \cos\left(\frac{\phi}{2}\right)$

  For $\phi_i = 0$, $A = 2a$ (Constructive interference)

  For constructive interference, path difference between two waves, $\Delta x = 0, \lambda, 2\lambda, ...n\lambda$

  For $\phi = \pi$, $A = 0$ (Destructive interference)

  For destructive interference, path difference between two waves,

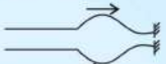
  $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, ...(2n - 1)\frac{\lambda}{2}$

4 REFLECTION OF WAVES

- Rigid Boundary – At rigid boundary wave suffer a phase change of π.

$y_i = a\sin(\omega t - kx)$

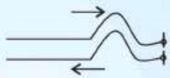
$y_r = -a\sin(\omega t + kx)$



- Open Boundary or Free boundary : At open boundary phase change is 0.

$y_i = a\sin(\omega t - kx)$

$y_r = a\sin(\omega t + kx)$



Standing Waves and Normal Modes

When two waves of same amplitude and of same frequency travel in opposite direction then resultant wave pattern from their superposition is called standing waves.

From open boundary.

$y_i(x, t) = a\sin(\omega t - kx)$,

$y_r(x, t) = a\sin(\omega t + kx)$

$y = y_i + y_r$

$y(x, t) = 2a\sin\omega t\cos kx$

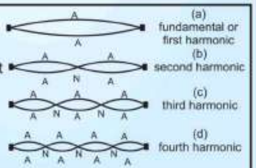
- The amplitude varies from point to point, but each element of string oscillate with same angular frequency (ω)
- **Nodes** – The point at which amplitude is zero or there is no motion called nodes. Distance between two consecutive nodes is $\lambda/2$.
- **Antinodes** – The points at which amplitude is maximum called antinodes. Distance between two consecutive antinodes is $\lambda/2$

Normal modes of stretched string Fixed At Both Ends

$L = \frac{n\lambda}{2}$, $n = 1, 2\ 3$

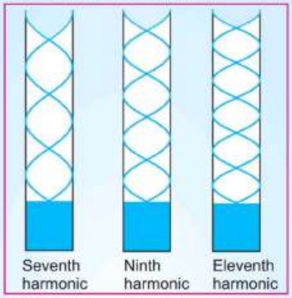
Frequencies of different modes

$\nu = \frac{nv}{2L}$, $n = 1, 2\ 3$–



5 NORMAL MODES OF ORGAN PIPES

Closed organ pipe



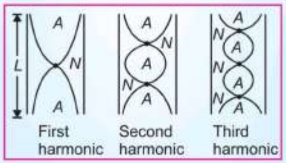
$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$: for $n = 0, 1, 2, 3,.....$

Possible wavelengths $\lambda = \frac{2L}{\left(n + \frac{1}{2}\right)}$

for $n = 0, 1, 2, 3$

Natural frequencies $\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$, for n or $= 0, 1, 2, 3$

Open Organ Pipe



$L = n\frac{\lambda}{2}$, for $n = 1, 2, 3...$

Wavelength of stationary wave, $\lambda = \frac{2L}{n}$, for $n = 1, 2, 3...$

Frequencies of different modes, $\nu = \frac{nv}{2L}$, for $n = 1, 2, 3...$

- A compression is reflected as compression from the closed end of the organ pipe and as rarefaction from the open end.
- A rarefaction is reflected as rarefaction from the closed end of the organ pipe and as compression from the open end.

6 BEATS

- When two harmonic sound waves of nearly same frequencies travel in the same direction then the intensity of resultant wave produced from their superposition increase and decrease continuously at same point with time. It is called beat formation.
- Two waves of angular frequencies ω_1 and ω_2 superimpose at , $x = 0$ at time t

$s_1 = a_1\cos\omega_1 t$, $s_2 = a_2\cos\omega_2 t$

from superposition, $s = s_1 + s_2$

$$s = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t.\cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

$\omega_b = \frac{(\omega_1 - \omega_2)}{2}$ and $\omega_a = \frac{(\omega_1 + \omega_2)}{2}$

Beat frequency, $\nu_{beat} = |\nu_1 - \nu_2|$

- We hear a waxing and warning of sound with frequency equal to difference between the frequencies of superposing waves.

7 DOPPLER EFFECT

Generally, if there is relative motion between a source(s) and observer then observed frequency will be other than real frequency. This apparent change in frequency is called Doppler effect.

- Both source and observer moving

$$\nu = \nu_0\left(\frac{v + v_0}{v + v_s}\right)$$

here v is the speed of sound through the medium, v_0 is the velocity of observer relative to the medium, and v_s is the source velocity relative to the medium. In using this formula, velocities in the direction O to S should be treated as positive and those opposite to it should be taken to be negative.

- When source and observer stationary and wind is blowing towards stationary observer with speed v_w, apparent wavelength

$\lambda_a = \frac{(v_0 - v_w)}{\nu}$

- When source is moving towards the stationary observer with medium at rest, apparent wavelength

$\lambda_a = \frac{(v - v_s)}{\nu}$